

Hello Future BC Calculus Students,

First off I'd like to say congratulations on making it to this level!

To Prepare for Advanced Placement Calculus BC, please complete the assigned problems. They will be due on the first day of school and will be collected. Please show your work on a separate paper. If you need to review concepts the major topics involved are Functions, Trigonometry, Limits, and Basic Derivatives. If you are stuck on any particular problem or topic, please make a note of it and ask within the first week of school. We will have a test on this material within the first week of school so you will have a small window of in class time to get your questions answered. Remember, all of this material in this packet is review.

Work on this in advance. **Do not try to do this last minute and in one night.**

I look forward to having you in class next year and please be ready to work hard, be challenged, and learn more than you have thus far!

Have a wonderful and relaxing summer!

There are only

$$\int_0^1 \frac{52x^{7/2} - 66x^{5/2} + 22x^{3/2}}{\sqrt{x}} dx$$

kinds of people  
in the world:

Those who know Calculus  
and those who don't.

1. Find an equation for the line that contains the points (2, -3) and (6, 9).

2. Find the value of  $y$  for which the line through  $A$  and  $B$  has the given slope  $m$ :  $A(-2, 3)$ ,  $B(4, y)$ ,

$$m = -\frac{2}{3}.$$

3. Find an equation for the line that contains the coordinate (5, 1) and is perpendicular to the line  $6x - 3y = 2$ .

For questions 4-9, let  $f(x) = \sqrt{x-3}$  and  $g(x) = x^2 + 5$ .

4.  $(f + g)(-3)$

5.  $(g - f)(6)$

6.  $g^{-1}(x)$

7.  $(g \circ f)(x)$

8.  $\frac{1}{f(x)}$

9.  $(f \circ g)(3)$

10. Algebraically find the inverse of  $y = \frac{3}{x-2} - 1$ .

For questions 11-19, simplify each expression completely.

11.  $\frac{\sqrt{x}}{x}$

12.  $e^{\ln 3}$

13.  $\ln 1$

14.  $\ln e^7$

15.  $\log_{\frac{1}{2}} 8$

16.  $e^{3 \ln x}$

17.  $\frac{4xy^{-2}}{12x^{\frac{-1}{3}}y^{-5}}$

18.  $27^{\frac{2}{3}}$

19.  $\frac{3x(x+1) - 2(2x+1)}{(x-1)^2}$

20. Rewrite  $\frac{1}{2}\ln(x-3) + \ln(x+2) - 6\ln x$  as a single logarithmic expression.

21. Solve for  $t$ :  $(1.045)^t = 2$

22. Solve for  $x$ :  $\log_5 x + \log_5(x-4) = 1$

23. Solve for  $x$ :  $27^{2x} = 9^{x-3}$

24. Solve for  $x$ :  $\ln(3x)^2 = 16$

25. Evaluate  $\log_2 5$  to the nearest thousandth.

26. Solve:  $x^3 + 3x^2 - 5x - 15 = 0$

27. Solve:  $x^4 - 9x^2 + 8 = 0$

28. Without using a calculator, find the exact value of  $\cos^{-1}\left(\cos\left(\frac{17\pi}{5}\right)\right)$ . Justify your answer.

For questions 29-34, find the exact values of each trigonometric function.

29.  $\sin\left(\frac{7\pi}{6}\right)$

30.  $\csc(60^\circ)$

31.  $\cos(120^\circ)$

32.  $\sec\left(-\frac{2\pi}{3}\right)$

33.  $\tan\left(\frac{\pi}{2}\right)$

34.  $\cot(-135^\circ)$

35. Simplify  $(\csc(x) - \tan(x))\sin(x)\cos(x)$

36. List the three Pythagorean Identities.

37. List the double angle formulas

a)  $\sin 2x =$

b)  $\cos 2x =$

38. List the sum and difference formulas.

a)  $\cos(\alpha \pm \beta) =$

b)  $\sin(\alpha \pm \beta) =$

39. Prove that  $\frac{\sin \theta}{1 - \cos \theta} + \frac{1 - \cos \theta}{\sin \theta} = 2 \csc \theta$

40. Prove that  $(\sin x + \cos x)^2 = 1 + \sin 2x$

41. For the solution of the equation  $2\sin^2 \theta = 1 - \sin \theta$  for  $0 \leq x < 2\pi$ .

42. Find the domain for  $k(x) = \sqrt{x^2 - 5x - 14}$ .

43. Determine all points of intersection for  $y = x^2 + 3x - 4$  and  $y = 5x + 11$ .

44. Find the points of intersection in the graphs of  $y = x - 1$  and  $y^2 = 2x + 6$ .

45. Use a graphing calculator to approximate all of the function's real zeros. Round your results to 3 decimal places.  $f(x) = 3x^6 - 5x^5 - 4x^3 + x^2 + x + 1$

46. Algebraically determine whether the function is even, odd, or neither:

a.  $f(x) = 2x^2 - 7$

b.  $f(x) = -4x^3 - 2x$

c.  $f(x) = 4x^2 - 4x + 4$

47. If  $f(x) = x^2 - 1$ , describe in words what the following would do to the graph of  $f(x)$ .

a.  $f(x) - 4$

b.  $f(x - 4)$

c.  $-f(x + 2)$

d.  $5f(x) + 3$

48. Let  $f(x) = \sqrt[3]{x+2}$  and  $g(x) = x^3 - 2$ . Which of the following are true?

I.  $g(x) = f^{-1}(x)$  for all values of  $x$ .

II.  $(f \circ g)(x) = 1$  for all values of  $x$ .

III. The function is one-to-one

49. For the function below, give the zeros (if non exist, write *none*), domain, range, VA's, HA's, and/or points of discontinuity (holes-as ordered pairs)if any exist. Also sketch the function's graph.

$$f(x) = \frac{x+3}{2x^2+5x-3}$$

**For questions 50-55, graph each function on the attached graph paper. Give its domain and range.**

50.  $y = -e^x$

51.  $y = |x+3| - 2$

52.  $y = 3 - 2\sin x$

53.  $y = 1 + \sqrt{x+2}$

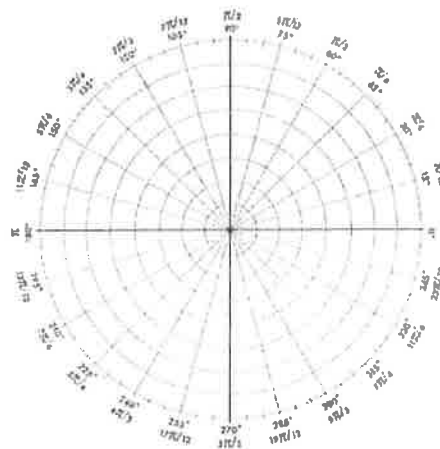
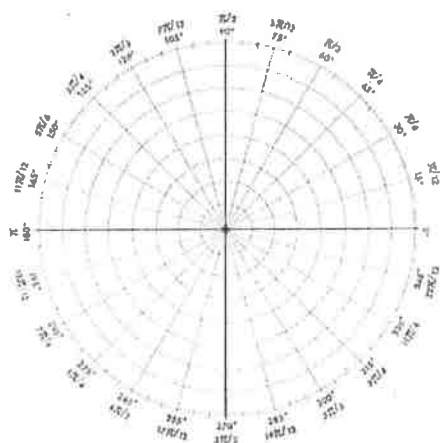
54.  $f(x) = \begin{cases} 1, & x \leq 0 \\ -1, & x > 0 \end{cases}$

55.  $f(x) = \begin{cases} 2x, & (-\infty, -1) \\ 2x^2, & (-1, 2) \\ -x+3, & (2, \infty) \end{cases}$

56. Please sketch the following polar curves.

$$r = -3 \sin \theta$$

$$r = 1 - 3 \cos \theta$$



57. Please convert the following polar coordinates to rectangular.

a.  $r = \frac{2}{5(\cos \theta - 3 \sin \theta)}$

b.  $r = 4 \sin t$

58. Please convert the following rectangular coordinates to polar

a.  $2x^2 + 2y^2 - x + y = 0$

b.  $x^2 + (y - 4)^2 = 16$

59. Find the sum of the following series:

a. The following geometric series  $\sum_{n=0}^{\infty} 3\left(-\frac{3}{4}\right)^n$

60. If  $\sum_{n=1}^{16} \frac{1}{n^2} =$

61. Which of the following series are the same. Justify your answer

i.  $\sum_{n=1}^{30} 3\left(\frac{1}{2}\right)^{n-1}$     ii.  $\sum_{n=0}^{29} 3\left(\frac{1}{2}\right)^n$     iii.  $3\sum_{n=0}^{29} \left(\frac{1}{2}\right)^n$     iv.  $3\sum_{n=1}^{30} \left(\frac{1}{2}\right)^n$

62.

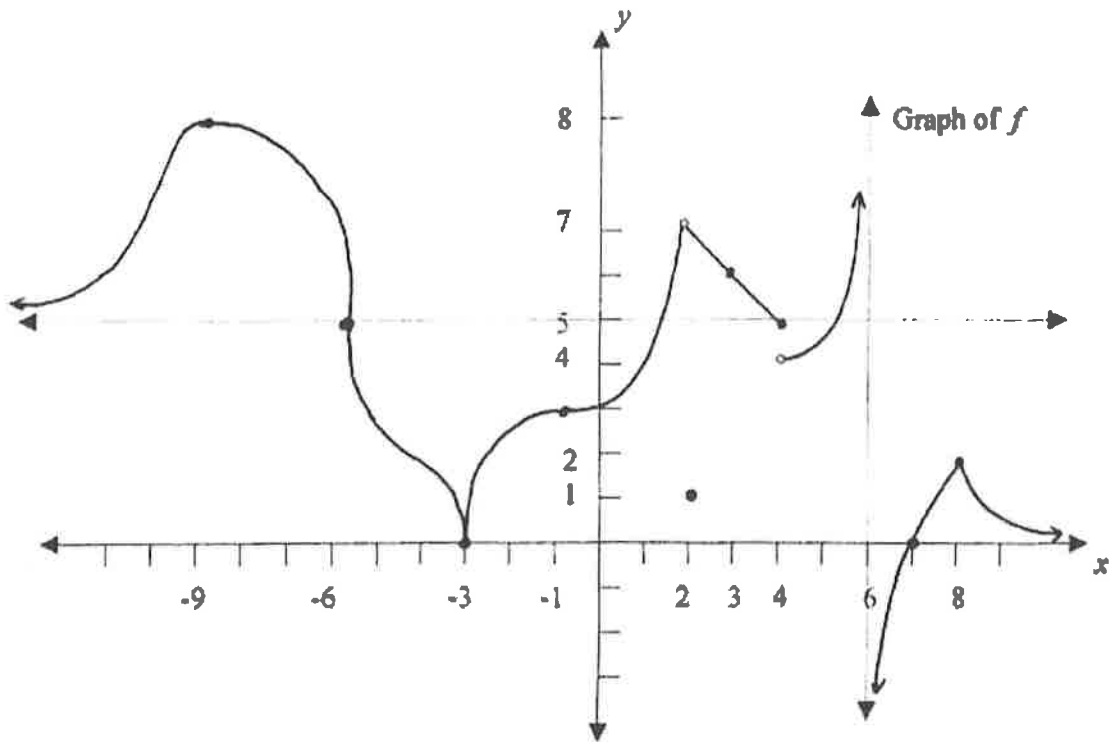
a) Write out the first 6 terms (starting at  $n = 0$ ) of the series with the following general term:

$$a_n = \frac{(-1)^{n+1} x^{2n}}{n!}$$

b) Given the following terms, find the general term for the series:

$$1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{48} + \frac{x^4}{384} + \dots$$

Use the graph below to answer questions 63-78



63.  $\lim_{x \rightarrow -3^-} f(x) =$

64.  $\lim_{x \rightarrow 4^+} f(x) =$

65.  $\lim_{x \rightarrow -3^+} f(x) =$

66.  $\lim_{x \rightarrow 4} f(x) =$

67.  $\lim_{x \rightarrow -3} f(x) =$

68.  $\lim_{x \rightarrow 6^-} f(x) =$

69.  $\lim_{x \rightarrow -6^+} f(x) =$

70.  $\lim_{x \rightarrow 2^-} f(x) =$

71.  $\lim_{x \rightarrow 6} f(x) =$

72.  $\lim_{x \rightarrow 2^+} f(x) =$

73.  $\lim_{x \rightarrow 2} f(x) =$

74.  $\lim_{x \rightarrow \infty} f(x) =$

75.  $\lim_{x \rightarrow 4^-} f(x) =$

76.  $\lim_{x \rightarrow \infty} f(x) =$

77.  $f(2) =$

78.  $f(3) =$



79. Find the  $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$

80. Find the  $\lim_{x \rightarrow 5} (2x^2 - 3x + 4)$

81. Find the  $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$

82. Find the  $\lim_{x \rightarrow 3} \frac{x^2 + x - 6}{x^2 - 9}$

83. Find the  $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$

84. Find the  $\lim_{t \rightarrow \infty} \frac{6t^2 + 5t}{(1-t)(2t-3)}$

85. Find the  $\lim_{x \rightarrow \infty} \cos x$

86. Find the  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

87. Find the  $\lim_{x \rightarrow \infty} \frac{2x^2}{5x^2 - 9x - 2}$

88. Find the  $\lim_{x \rightarrow \infty} \frac{x^2 + x}{3 - x}$

89. Find the  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$

90. Find the slope of the tangent line to the graph of  $g(x) = x^2 - 4$  at the point  $(1, -3)$ .91. Find an equation of the tangent line to the hyperbola  $y = \frac{3}{x}$  at the point  $(3, 1)$ .92. Find an equation of the tangent line to the graph of  $f(x) = x^2 - 8x + 9$  at the point  $(3, -6)$ .

**For questions 93-95, find the x-values (if any) at which  $f$  is not continuous. Which of the discontinuities are removable.**

93.  $f(x) = \frac{x}{x^2 - x}$

94.  $f(x) = 3x - \cos x$

95.  $f(x) = \frac{x+2}{x^2 - 3x - 10}$

For questions 96-116, find the derivative of each given function.

96.  $f(x) = x^{\frac{2}{5}}$

97.  $f(x) = x^3 - x^2 + 2x$

98.  $y = 5e^x + 3$

99.  $f(x) = \sqrt[3]{x}$

100.  $g(x) = \frac{\sqrt{10}}{x^7}$

101.  $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$

102.  $f(x) = x^2 e^x$

103.  $f(x) = \frac{e^x}{x^2}$

104.  $f(x) = \frac{x}{(8-3x)}$

\* 105.  $\tan \sqrt{1-x}$

106.  $y = \sin x + \cos x$

107.  $y = x^2 \cos x$

108.  $f(x) = \cos x - 2 \tan x$

\* 109.  $f(x) = 2 \sin(x^2) + \cos^3 x$

\* 110.  $g(x) = (x^3 - 1)^{100}$

\* 111.  $y = \sqrt{x^2 - 7x}$

\* 112.  $y = (2x+1)^5 (x^3 - x + 1)^4$

\* 113.  $y = \ln(x^3 + 1)$

114. Find  $y'$  of  $y = \frac{e^x \cdot \sin x}{\sqrt{x}}$

115. If  $f(x) = x^2 - 2e^x$ , find the value of  $f'(1)$

116. If  $f(x) = 2x^2 - x^3$ , find  $f'(x)$ ,  $f''(x)$ , and  $f'''(x)$

\* These involve chain rule.